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A measure for triaxiality from K (shape) invariants

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Abstract. We show that the cubic shape invariant K_3, which provides a measure for triaxiality for β-rigid nuclei, can be obtained from a small number of observables. This affords approximations which have been tested to hold within a few percent in the rigid triaxial rotor model and the interacting boson model.

PACS. 21.10.Ky Electromagnetic moments – 21.60.Ev Collective models – 21.60.Fw Models based on group theory

1 Introduction

The deformation parameters β and γ from the geometrical model are not easy to define in all cases, *e.g.*, in vibrational nuclei. They are of course model dependent and usually do not take fluctuations into account. An alternative approach to nuclear deformation is given by quadrupole shape invariants [1,2]. These shape invariants can be considered as the "real" shape parameters, as they are observables, and hence do not involve any model input. However, exact values can in general only be obtained from large (complete) sets of $E2$ matrix elements, which are rarely available, for a small set of stable nuclei. The aim of the current work is to show that the shape parameter K_3, which is related to triaxiality, can be obtained with good accuracy from only a few experimental observables. While triaxiality is discussed also for excited states and bands, *e.g.*, in terms of chirality [3] or wobbling [4], the following discussion is restricted to triaxiality in the ground state of even-even nuclei.

Quadrupole shape invariants are defined by [5]

$$K_n = q_n/\left(q_2^{n/2}\right), \tag{1}$$

with higher-order moments of the quadrupole operator in a given state, in our case the ground state, of the type

$$q_n = \alpha_n \, \langle 0_1^+ | \, [\, \underbrace{Q \ldots Q}_{n} \,]^{(0)} \, |0_1^+\rangle \,, \tag{2}$$

with geometrical factors α_n and using tensor coupling for the quadrupole operators Q. In analogy to the geometrical model, where β and γ correspond to a (rigid) minimum in the potential, effective deformation parameters can be defined as averages in the ground state,

$$q_2 = \left(\frac{3ZeR^2}{4\pi}\right)^2 \langle\beta^2\rangle \equiv \left(\frac{3ZeR^2}{4\pi}\right)^2 {\beta_{\mathrm{eff}}}^2, \tag{3}$$

and

$$K_3 = -\frac{\langle\beta^3\cos 3\gamma\rangle}{\langle\beta^2\rangle^{3/2}} \equiv -\cos(3\gamma_{\mathrm{eff}}), \tag{4}$$

with nuclear radius R, proton number Z and charge e. K_4 and K_6 give measures for fluctuations in β and γ in the non-rigid case. While, for the rigid rotor, γ_{eff} is equal to the geometrical γ-value, it provides a measure for effective triaxiality also for vibrational or γ-soft nuclei. However, we note that the K-parameters are in general not equivalent to the geometrical deformation parameters. This is due to fluctuations in β and γ which are incorporated in the averages in the ground state (compare eqs. (2)–(4)). Due to averaging over $\beta^3\cos 3\gamma$, eq. (4) gives an effective γ-deformation related to the geometrical model only if β is rigid. This is the case for nuclei transitional between the well-deformed and the triaxial (γ-soft) rotor. On the other hand, if γ is rigid, K_3 reflects the softness in β, which can be expected for nuclei transitional between the well-deformed rotor and the vibrator for $\gamma = 0°$.

2 Results

In general, K_3 involves a large number of $E2$ matrix elements. Applying the Q-phonon scheme [6] and its $\Delta Q = 1$ selection rule for $E2$ transitions, the number of needed matrix elements reduces drastically and K_3 can be written in terms of the quadrupole moment $Q(2_1^+)$ only. However, from calculations in the interacting boson model (IBM-1) and the rigid triaxial rotor model (RTRM) it is seen that this approach leads to large deviations from the exact value of K_3 in transitional regions.

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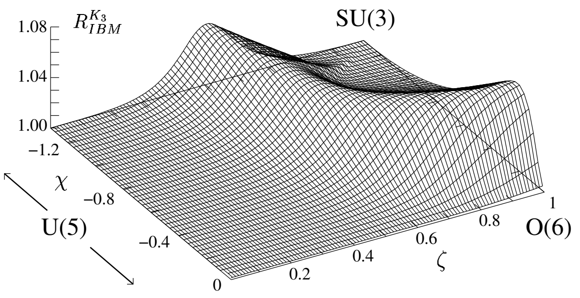

Fig. 1. $R^{K_3}_{\mathrm{IBM}}$, calculated over the IBM-1 symmetry space.

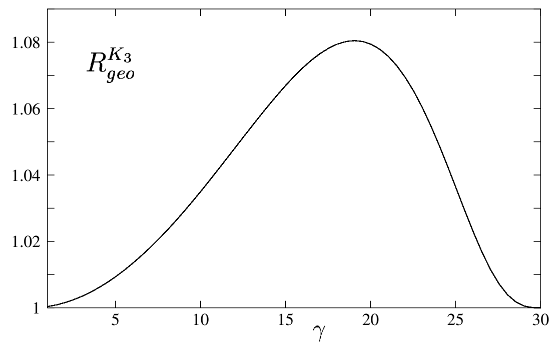

Fig. 2. $R^{K_3}_{\mathrm{geo}}$, calculated within the RTRM for $\gamma \in [0, 30]$.

In a second approximation, introducing $B(E2; 2^+_1 \to 2^+_1) = 35/(32\pi) \cdot Q(2^+_1)^2$ [7], and allowing one matrix element with $\Delta Q = 2$, we derive an approximation [8]

$$K_3^{\mathrm{appr}} = \sqrt{\frac{7}{10}}\ \mathrm{sign}\left(Q(2^+_1)\right)\left(\sqrt{\frac{B(E2;2^+_1\to 2^+_1)}{B(E2;2^+_1\to 0^+_1)}} -2\frac{\sqrt{B(E2;2^+_2\to 0^+_1)\cdot B(E2;2^+_2\to 2^+_1)}}{B(E2;2^+_1\to 0^+_1)}\right), \quad (5)$$

involving four $B(E2)$ values. The derivation of eq. (5) incorporates a sign relation between the four involved matrix elements [9], namely

$$\mathrm{sign}\left(\langle 2^+_1||Q||2^+_1\rangle\right) = -\,\mathrm{sign}\left(\langle 0^+_1||Q||2^+_2\rangle\langle 2^+_2||Q||2^+_1\rangle\langle 2^+_1||Q||0^+_1\rangle\right). \quad (6)$$

Equation (5) gives a well defined way for deriving an approximate K_3 from data. In order to get an estimate on the error resulting from the truncation, the validity of the approximation needs to be tested within models, which has been done within the IBM-1 and the RTRM. Deviations of the value of K_3^{appr} given by eq. (5) from the exact K_3 are small as shown in fig. 1. Herein, the ratio

$$R^{K_3}_{\mathrm{IBM}} = \frac{1+|K_3^{\mathrm{appr}}|}{1+|K_3|} \quad (7)$$

has been calculated over the whole parameter space of the two-parameter IBM-1 Hamiltonian

$$H_{\mathrm{IBM}} = (1-\zeta)\ n_d - \frac{\zeta}{4N}\ Q^\chi \cdot Q^\chi \quad (8)$$

by variation of ζ and χ, including the vibrator ($U(5)$), the well-deformed rotor ($SU(3)$), and the γ-soft rotor ($O(6)$) limits. Calculations within the RTRM show the same quality of our approximation as depicted in fig. 2, where $R^{K_3}_{\mathrm{geo}}$ is defined in analogy to eq. (7).

Note that the absolute value of the quadrupole moment can be calculated from the 3-$B(E2)$-relation

$$B\big(E2; 2^+_1 \to 2^+_1\big) = B\big(E2; 4^+_1 \to 2^+_1\big) - B\big(E2; 2^+_2 \to 2^+_1\big), \quad (9)$$

which was derived in a similar way in [7].

Table 1. The approximative shape invariant K_3^{appr} for transitional Os isotopes. Effective γ- and β-deformation parameters are given in the last two columns.

	K_3^{appr}	γ_{eff}	β_{eff}
^{188}Os	−0.63(5)	17(3)	0.185
^{190}Os	−0.35(9)	23(3)	0.177
^{192}Os	−0.3(1)	25(2)	0.167

Exemplarily, table 1 gives K_3^{appr} and γ_{eff}, derived from eqs. (5) and (4), respectively, for Os isotopes transitional between the rigid axially symmetric rotor and the γ-soft rotor. Data has been taken from the nuclear data sheets and stems mostly from D. Cline and co-workers, who made large sets of $E2$ matrix elements available. Included in table 1 are the calculated values of β_{eff} from eq. (3) where, using the same truncation as for K_3^{appr}, q_2 can be approximated by

$$q_2^{\mathrm{appr}} = B\big(E2; 0^+_1 \to 2^+_1\big). \quad (10)$$

It is seen that β-deformation decreases while the value of γ rises towards 30°, which is the limit of maximum (soft) triaxiality.

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